

Occurrence of instability through the protostellar accretion disks by landing of low-mass condensations

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ABSTRACT

Low-mass condensations (LMCs) are observed inside the envelope of the collapsing molecular cloud cores. In this research, we investigate the effects of landing LMCs for occurrence of instability through the protostellar accretion disks. We consider some regions of the disk where duration of infalling and landing of the LMCs are shorter than the orbital period. In this way, we can consider the landing LMCs as density bumps and grooves in the azimuthal direction of an initial thin axisymmetric steady state self-gravitating protostellar accretion disk (nearly Keplerian). Using the linear effects of the bump quantities, we obtain a characteristic equation for growth/decay rate of bumps; we numerically solve it to find occurrence of instability. We also evaluate the minimum-growth-time-scale (MGTS) and the enhanced mass accretion rate. The results show that infalling and landing of the LMCs in the inner regions of the protostellar accretion disks can cause faster unstable modes and less enhanced accretion rates relative to the outer regions. Also, more fragmentation of landed LMCs in the azimuthal direction have less chance for instability, and then can produce more values of enhanced mass accretion rate.

Subject headings: accretion disks – instabilities – planets and satellites: formation

1. Introduction

Dense cores through the molecular clouds are nurseries of protostellar accretion disks. There has been a lot of observations which show that the structure of these dense cores are clumpy with small masses and sizes. For example, Langer et al. (1995) observed low mass

condensations (LMCs) in the core D of Taurus Molecular Cloud 1 in the regime of 0.007-0.021 pc and $0.01 - 0.15 M_{\odot}$. Also, we can refer to the discovery of very low luminosity objects by the Spitzer From Molecular Cores to Planet-Forming disks’ (c2d) project (Lee et al. 2009, Dunham et al. 2014) or the results gained by Launhardt et al. (2010), in which they found that at least two-thirds of 32 studied isolated star-forming cores show evidence of forming multiple stars. For other observations, we can refer to millimeter and submillimeter observations toward two prestellar cores SM1 and B2-N5, in active cluster forming regions, that showed the presence of small scale objects inside the prestellar cores with masses in the range of $10^{-2} - 10^{-1} M_{\odot}$ and sizes of a few hundred AU (Nakamura et al. 2012). These observations, and also the works of Pirogov & Zinchenko (2008) and Tachihara (2013), can be used as a witness to the existence of LMCs in the molecular cloud cores. Since the masses of these condensations are small and the molecular cloud cores are almost quiescent, gravitational instability and turbulent can not be considered as the responsible mechanisms for clumping through the cores. Nejad-Asghar(2011a) showed that thermal instability may be considered as an important mechanism for formation of the LMCs in the envelope of the molecular cloud cores.

Initial rotation of the collapsing dense cores can lead to the formation of an accretion disk, in which may ultimately be ended to the formation of the proto-planetary entities (e.g., Stahler 2004, Hartmann 2009). There are two known mechanisms for the formation of the condense proto-planetary entities through the protostellar accretion disks: the core accretion (Goldreich & Ward 1973, Miguel & Brunini 2009), and the disk instability (Toomre 1964, Boss 1997, Zhu et.al. 2012). The core accretion mechanism occurs from the collision and coagulation of dusty solid particles into gradually larger bodies until a massive enough proto-planet is formed. In the instability model, a disk around a protostar may be fragmented by instability and these pieces may become proto-planetary entities. In this paper, we neglect the dusty solid particles and focus on the latter mechanism. In order to occur the instability and disk fragmentation, some physical conditions must be satisfied. Two criteria are mostly used to discuss whether a protostellar disk is likely to fragment. The first is Toomre’s stability criterion (Toomre 1964): $Q = \frac{c_s \kappa}{\pi G \Sigma} > 1$, where c_s is the sound speed, κ is the local epicyclic frequency, and Σ is the disk surface density. If $Q < 1$, the disk may be gravitationally unstable (e.g., Binney and Tremaine 2008). The second criterion is Gammie’s cooling criterion (Gammie 2001): $t_{cool} < \frac{1}{\Omega}$, where Ω is the angular velocity. Gammie suggested that the disk must be quickly cooled until the disk can fragment and gravitationally bound gaseous objects form.

In this research, we consider the phase of the disk, in which the matters from envelope (especially LMCs) of the collapsing molecular cloud core are still infalling onto the disk. The high velocity of infalling LMCs from the envelope of the cores onto the accretion disk, leads

to formation of the shocked waves (Mendoza 2009, Nejad-Asghar 2011b). If the timescales of cooling and relaxing of the shocked waves are much shorter than the orbital period, we expect to form density bumps and grooves through the protostellar accretion disks. Nejad-Asghar(2011c) showed that timescales for formation shock waves and cooling down to form density bumps on the accretion disks, are about few hundred years. In this way, we consider some regions of the protostellar accretion disk where infalling and landing of the LMCs on there can lead to formation of density bumps and grooves in a timescales much shorter than orbital period. These density bumps change the surface density of the protostellar accretion disks and can affect on the gravitational potential and pressure gradient, so that the disk may be unstable. In this research, we consider a thin axisymmetric steady state self-gravitating accretion disk in cylindrical coordinate and investigate occurrence of instability caused by these density bumps and grooves. Formulation of the problem is given in the section 2, and the results with some concluding remarks are presented in the section 3.

2. Formulation of the problem

The fundamental equations governing thin rotating disk with $\mathbf{u} = u_R\hat{r} + u_\varphi\hat{\varphi}$, in the cylindrical polar coordinates (R, φ) are (e.g., Clark & Carswell 2007)

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R}(R \Sigma u_R) + \frac{1}{R} \frac{\partial}{\partial \varphi}(\Sigma u_\varphi) = 0, \quad (1)$$

$$\frac{\partial u_R}{\partial t} + u_R \frac{\partial u_R}{\partial R} + \frac{u_\varphi}{R} \frac{\partial u_R}{\partial \varphi} - \frac{u_\varphi^2}{R} = -\frac{\partial \Phi}{\partial R} - \frac{1}{\Sigma} \frac{\partial P}{\partial R}, \quad (2)$$

$$\frac{\partial u_\varphi}{\partial t} + u_R \frac{\partial u_\varphi}{\partial R} + \frac{u_\varphi}{R} \frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\varphi u_R}{R} = -\frac{1}{R} \frac{\partial \Phi}{\partial \varphi} - \frac{1}{R \Sigma} \frac{\partial P}{\partial \varphi}, \quad (3)$$

where Σ and Φ are the surface density and the gravitational potential, respectively, and P is the gas pressure that is given by the equation of state

$$P = \frac{k_B}{\mu m_H} \Sigma T, \quad (4)$$

where k_B , μ , m_H , and T are the Boltzman constant, mean molecular mass, hydrogen atomic mass, and the temperature of the disk, respectively. In this paper, we aim to study the effect of the landing LMCs on the instability of the disk with initial mass accretion rate less than $10^{-6} M_\odot yr^{-1}$. Since infalling and landing of the LMCs on the accretion disk, produces shock waves and viscous heating, we consider the thermal effect of viscosity, while its dynamical effects in the momentum equations (2) and (3) are ignored. It should be noted that to analyze the dynamical effects in low-rate accretion flows ($\lesssim 10^{-6} M_\odot yr^{-1}$), we assume the viscous

terms are negligible compared to the potential and pressure gradient. Also, we suppose the disk to be in thermal equilibrium, so that energy sources and sinks are in local balance; the heating sources include viscous dissipation and external irradiation intercepted by the disk, and the energy sink is affected by radiative cooling. Here, we do not directly evaluate the temperature and surface density of the viscous heated accretion disks. Instead, we use a low-rate accretion disk model with $\Sigma_0(R)$ and $T_0(R)$ as recently obtained by the work of Rafikov (2015).

We consider a system including a central protostar with mass $M_* = M_\odot$ which is surrounded by a self-gravitating gaseous disk (initial smooth accretion disk). Also, we assume the disk is initially in steady state with axisymmetric behavior ($\frac{\partial}{\partial t} = 0$ & $\frac{\partial}{\partial \varphi} = 0$). Also, in the low-rate accretion flows, the radial velocity is smaller than the azimuthal velocity, thus, the radial velocity is supposed to be negligible, i.e., $u_R = 0$. In this steady axisymmetric state, the equations (1)-(3) reduce to

$$\frac{u_\varphi^2}{R} = -g + \frac{1}{\Sigma_0} \frac{dP_0}{dR}, \quad (5)$$

where, $g = -\frac{\partial \Phi}{\partial R}$ is the gravitational field. We separate the gravitational field to the central mass contribution, $g^c = -\frac{GM_*}{R^2}$, and the disk contribution, which depends on the surface density through the Poissons equation (e.g., Hure & Pierens 2005) given by

$$g^d(R) = -G \int_{R_{in}}^{R_{out}} \sqrt{\frac{R'}{R}} \frac{\kappa}{R} \Sigma_0(R') \left[K(\kappa) - \frac{E(\kappa)}{\varpi} \right] dR'. \quad (6)$$

where, $K(\kappa)$ and $E(\kappa)$ are the complete elliptic integrals of the first and second kinds, respectively, $\kappa = 2\sqrt{\frac{R'R}{R'+R}}$ is their modulus, and $\varpi = \frac{(R'-R)}{(R'+R)}$. Substituting $u_\varphi = R\Omega(R)$ into equation (5) and using equation (4), the angular velocity is given by

$$\Omega_0 = \sqrt{\Omega_K^2 + \frac{1}{R}(-g^d + \frac{k_B}{\mu m_H} \frac{1}{\Sigma_0} \frac{d}{dR}(\Sigma_0 T_0))}, \quad (7)$$

where, $\Omega_K = \sqrt{\frac{GM_*}{R^3}}$ is the Keplerian angular velocity.

We choose the dimensionless scales for length, mass, time, and temperature as $[R] = 100AU$, $[M] = 2.9 \times 10^{25}kg$, $[t] = 50yr$, and $[T] = 10K$, respectively, so that, $[\Sigma] = 0.7[M][R]^{-2} = 0.09kg/m^2$, $G = 1.23 \times 10^{-6}[R]^3[M]^{-1}[t]^{-2}$, and $\frac{k_B}{\mu m_H} = 5.73 \times 10^{-4}[R]^2[T]^{-1}[t]^{-2}$ with choosing $\mu = 1.5$. With these scales, we consider a model for initial smooth low-rate accretion disk ($\approx 7 \times 10^{-7}M_\odot yr^{-1}$) with surface density and temperature profiles as

$$\Sigma_0(R) = 1995.2R^{-1.7}, \quad T_0(R) = 4.3R^{-1.3}, \quad (8)$$

respectively (Rafikov 2015). Fig. 1 shows the radial profiles of the surface density Σ_0 , temperature T_0 , Toomre parameter $Q_0 = c_s \Omega_0 / \pi G \Sigma_0$, orbital period $2\pi / \Omega_0$, and relative difference between the angular velocity with its Keplerian one $(\Omega_K - \Omega_0) / \Omega_K$. We can see that the disk rotation is nearly Keplerian. According to the work of Nejad-Asghar (2011c) for the timescales for formation of density bumps (i.e., few hundred years), we consider regions of the protostellar disk (i.e., radius between 0.4 to $3[R]$), which the orbital period is greater than these timescales.

Here, we assume that density bumps are small and can cause to produce the small quantities: gravitational potential (Φ_1), pressure (P_1), radial velocity (u_{R1}), and azimuthal velocity ($u_{\phi 1}$). For finding the relation between these quantities to the density bump, we consider $\Sigma = \Sigma_0 + \Sigma_1$, $u_R = u_{R1}$, $u_\varphi = u_{\varphi 0} + u_{\varphi 1}$, $P = P_0 + P_1$, and $\Phi = \Phi_0 + \Phi_1$, where the subscripts "0" and "1" represent the initial steady state accretion disk and bump quantities, respectively. Substituting these relations into equations (1)-(3), in the first-order approximation, we have

$$\frac{\partial \Sigma_1}{\partial t} + \frac{1}{R} \frac{\partial}{\partial R} (R \Sigma_0 u_{R1}) + \frac{\Sigma_0}{R} \frac{\partial}{\partial \varphi} (u_{\varphi 1}) + \Omega_0 \frac{\partial}{\partial \varphi} (\Sigma_1) = 0, \quad (9)$$

$$\Sigma_0 \frac{\partial u_{R1}}{\partial t} + \Sigma_0 \Omega_0 \frac{\partial u_{R1}}{\partial \varphi} - 2 \Sigma_0 \Omega_0 u_{\varphi 1} - \Sigma_1 R \Omega_0^2 = -\Sigma_0 \frac{\partial \Phi_1}{\partial R} - \Sigma_1 \frac{\partial \Phi_0}{\partial R} - \frac{\partial P_1}{\partial R}, \quad (10)$$

$$\Sigma_0 \frac{\partial u_{\varphi 1}}{\partial t} + \Sigma_0 u_{R1} \frac{\partial u_{\varphi 0}}{\partial R} + \frac{1}{R} \Sigma_0 u_{\varphi 0} \frac{\partial u_{\varphi 1}}{\partial \varphi} + \Sigma_0 \frac{u_{\varphi 0} u_{\varphi 1}}{R} = -\frac{\Sigma_0}{R} \frac{\partial \Phi_1}{\partial \varphi} - \frac{1}{R} \frac{\partial P_1}{\partial \varphi}. \quad (11)$$

The azimuthal and time dependency of the bump quantities can be expanded by the Fourier terms as follows

$$\begin{aligned} u_{R1} &= \text{Re}[u_{Ra}(R) \exp(i(m\varphi - \omega t))], & u_{\varphi 1} &= \text{Re}[u_{\varphi a}(R) \exp(i(m\varphi - \omega t))], \\ \Sigma_1 &= \text{Re}[\Sigma_a(R) \exp(i(m\varphi - \omega t))], & \Phi_1 &= \text{Re}[\Phi_a(R) \exp(i(m\varphi - \omega t))], \\ P_1 &= \text{Re}[P_a(R) \exp(i(m\varphi - \omega t))], \end{aligned} \quad (12)$$

where, m is the azimuthal mode number and ω is a complex number which its imaginary part indicates growth/decay rate according to its sign. In general, ω must be a function of bump position in the accretion disk, but for simplicity, we consider it as a constant value at the peak-point of the bump. Here, we assume that infalling and landing of the LMCs lead to formation Gaussian density bumps at the timescale shorter than orbital period, as

$$\Sigma_a(R) = \Sigma'_a \exp(-4(\frac{R - R_1}{\Delta_1})^2), \quad (13)$$

where Σ'_a , R_1 , and Δ_1 are the amplitude, peak-point position, and e-folding width of the bump, respectively. Also, we assume that the bumped gravitational potential $\Phi_a(R)$ is

related to the density bump via the Poisson equation (Hure & Pierens 2005)

$$\Phi_a(R) = -2G \int_{R_{in}}^{R_{out}} \sqrt{\frac{R'}{R}} \kappa \Sigma_a(R') K(\kappa) dR', \quad (14)$$

and the bumped pressure is given by $P_a = \frac{k_B}{\mu m_H} \Sigma_a T_0$.

Now, considering equations (12), the equations (9)-(11) reduce to

$$i(m\Omega_0 - \omega)\Sigma_a + \frac{1}{R} \frac{d}{dR}(R\Sigma_0 u_{Ra}) + \frac{im\Sigma_0}{R} u_{\varphi a} = 0, \quad (15)$$

$$i(m\Omega_0 - \omega)\Sigma_0 u_{Ra} + \left(\frac{d\Phi_0}{dR} - R\Omega_0^2\right)\Sigma_a - 2\Sigma_0\Omega_0 u_{\varphi a} = -\Sigma_0 \frac{d\Phi_a}{dR} - \frac{dP_a}{dR}, \quad (16)$$

$$i(m\Omega_0 - \omega)\Sigma_0 u_{\varphi a} + \Sigma_0 \frac{d}{dR}(R\Omega_0 + \Sigma_0\Omega_0)u_{Ra} = -i\frac{m\Sigma_0}{R}\Phi_a - i\frac{m}{R}P_a. \quad (17)$$

By merging equations (16) and (17), the amplitude of radial and azimuthal components of the bumped velocity are obtained as follows

$$u_{Ra} = -i \frac{\left((R\Omega_0^2 - \frac{d\Phi_0}{dR} - \frac{1}{\mu} \frac{dT_0}{dR} - 2\frac{m\Omega_0 T_0}{R\mu} \frac{1}{(m\Omega_0 - \omega)})\Sigma_a - \Sigma_0 \frac{d\Phi_a}{dR} - \frac{T_0}{\mu} \frac{d\Sigma_a}{dR} - 2\frac{m\Omega_0 \Sigma_0}{R} \frac{\Phi_a}{(m\Omega_0 - \omega)}\right)}{\Sigma_0((m\Omega_0 - \omega) - 2\Sigma_0\Omega_0 \frac{(R\frac{d\Omega_0}{dR} + 2\Omega_0)}{(m\Omega_0 - \omega)})}, \quad (18)$$

$$u_{\varphi a} = i \frac{1}{(m\Omega_0 - \omega)\Sigma_0} \left(\Sigma_0 \frac{d}{dR}(R\Omega_0 + \Sigma_0\Omega_0)u_{Ra} + i\frac{m\Sigma_0}{R}\Phi_a + i\frac{m}{R}P_a\right). \quad (19)$$

Now, by substitution u_{Ra} and $u_{\varphi a}$ into equation (15), we obtain a characteristic polynomial equation as

$$a_6\omega^6 + a_5\omega^5 + a_4\omega^4 + a_3\omega^3 + a_2\omega^2 + a_1\omega + a_0 = 0, \quad (20)$$

where the coefficients $a_0, a_1, a_2, a_3, a_4, a_5$, and a_6 depend on the radial profile of surface density, temperature and angular velocity of our chosen model for smooth protostellar accretion disk (i.e., Σ_0, T_0 and Ω_0), on the density bump parameters (i.e., Σ'_a and Δ_1 and its peak-point position R_1), and especially on azimuthal mode number m , which represents initial fragmentation of bump quantities (number of bumps and grooves) across the azimuthal direction at radius R_1 of the disk.

3. Numerical results and conclusion

In this paper, a phase of collapsing molecular cloud cores was surveyed, in which an accretion disk around a protostar was formed. In this phase, the infalling and landing of LMCs, which were preformed in the core envelope, could lead to change the dynamics of the

protostellar disk. Infalling and landing of LMCs on the protostellar accretion disk produces shocked waves, which if they cool rapidly in a timescale shorter than the orbital period, can lead to formation of density bumps and grooves through initial smooth disk. Assuming the bump quantities are small in amplitude, and using the linear analysis on the basic equations, we obtained the characteristic equation (20).

The characteristic equation (20) is a polynomial function of ω , and its coefficients are evaluated at the peak point position of the density bumps, R_1 . We use the Laguerre's method (Press et.al 1992) to find the roots of this polynomial equation. The positive/negative sign of the imaginary part of ω represents the growth/damping rate. We locate the density bumps at different radii of the disk, and find the roots of the characteristic equation. According to the obtained imaginary part of the root, the instability and growth rates of the bumped accretion disk at different radii can be found. If at each radius, we obtain many roots with positive imaginary parts, we consider maximum value of them as maximum growth rate. Inverse of the maximum growth rate, $\frac{1}{\max(\Im(\omega))}$, is the minimum-growth-time-scale (MGTS), which represent the growth time of the bump quantities and formation of e-folded density clumping through protostellar accretion disks.

Considering density bumps with parameters $\Sigma'_a = 0.01\Sigma_0$ and $\Delta_1 = 0.1[R] = 10AU$, the normalized MGTS by the orbital period, as a function of the position of the pick-point of bump, R_1 , are shown in the Fig. 2, for different values of azimuthal mode number $m = 1, 10, 15$, and 20. As can be seen, increasing the azimuthal number m , causes the normalized MGTS to increase. Increasing m would produce more values of bumps and grooves in the azimuthal direction. As a result, if infalling and landing of the LMCs would lead to less fragmentation of the bumps (i.e., smaller values of m), MGTSs decrease and instability would happen with more chance. Moreover, landing of LMCs in the outer parts of the accretion disk lead to increasing of MGTSs. Therefore, bumping in the denser parts of the protostellar disk (i.e., smaller values of R) would cause the instability to happen faster. From the results, it can be said that bumping closer to the protostar (i.e., smaller values of R) would cause more instability, and thus increases the probability of formation of proto-planetary entities over there.

The enhanced mass accretion rates, $\dot{M} = 2\pi R\Sigma_0|u_{Ra}|$, relative to the initial mass accretion rate $\dot{M}_0 \approx 7 \times 10^{-7} M_\odot yr^{-1}$, are shown in the Fig. 3, as a function of the position of the pick-point of bumps, for $m = 1, 10, 15$, and 20. It is seen that, mass accretion rate is smaller in the inner regions of the protostellar disk where the normalized MGTSs have smaller values. In regions where instability would be more/less likely to occur (i.e., smaller/greater values of MGTS), the rate of mass accretion onto the central mass object will be reduced/incremented via increasing/decreasing of coagulation of matter and growth of condensations. In the other

words, when the MGTS is short, formed bump can grow and get bigger. Thus, the matters will be gathered in the place of the formed density bumps and cannot fall down on the central mass, therefore enhanced mass accretion rate decreases, and vice versa. Also, the results of Fig. 3 show that the enhanced mass accretion rate increases with increasing the azimuthal mode number m . Thus, more fragmentation of landed LMCs in the azimuthal direction (i.e., greater values of m) have less chance for instability, and then can produce more values of enhanced mass accretion rate. In other words, smaller values of azimuthal mode number (i.e., smaller numbers of bumps and grooves across the azimuthal direction), in the inner regions of the protostellar accretion disks, have more chance for instability and growing of condensations to form proto-planetary entities.

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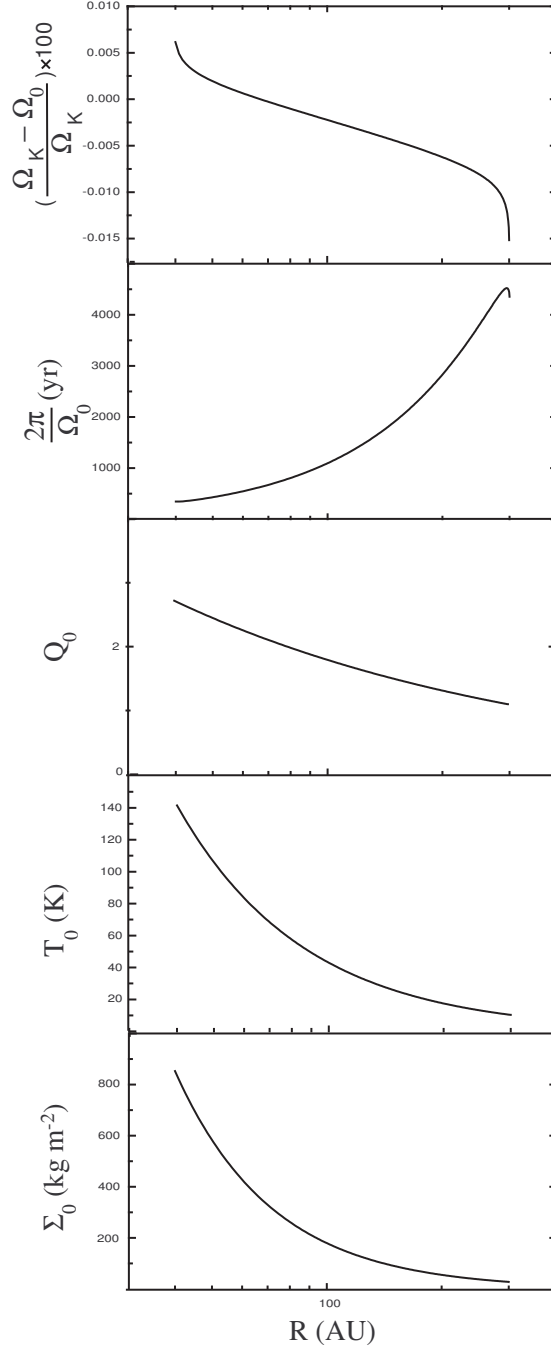


Fig. 1.— Radial profiles of surface density Σ_0 , temperature T_0 , Toomre parameter $Q_0 = c_s \Omega_0 / \pi G \Sigma_0$, orbital period $2\pi / \Omega_0$, and relative difference between the angular velocity with its Keplerian one $(\Omega_K - \Omega_0) / \Omega_K$, for initial protostellar accretion disk model described by equation (8).

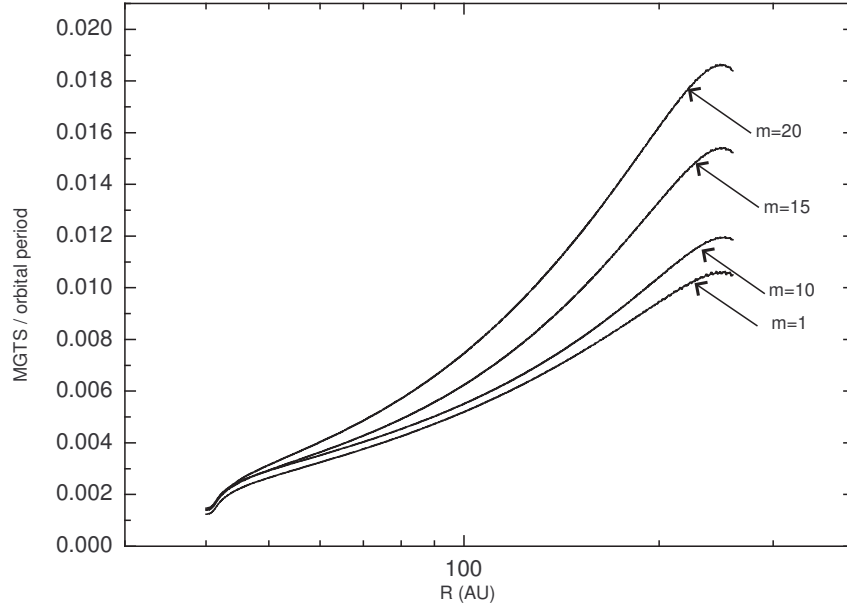


Fig. 2.— The normalize MGTS by the orbital period, as a function of the position of the pick-point of bumps, for azimuthal mode numbers $m = 1, 10, 15$, and 20 . The parameters of density bump are chosen as $\Sigma'_a = 0.01\Sigma_0$ and $\Delta_1 = 0.1[R] = 10AU$.

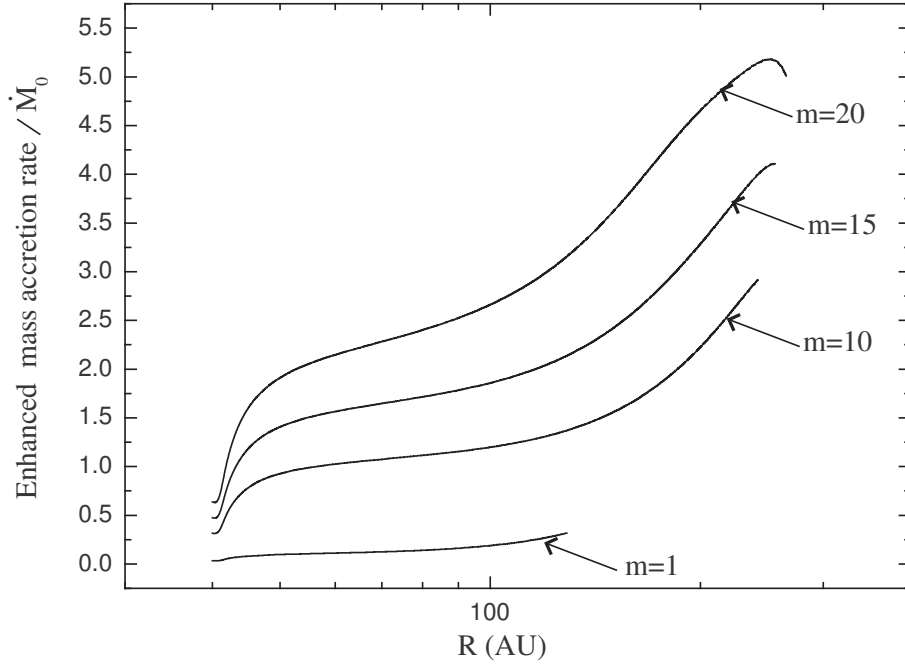


Fig. 3.— The ratio of enhanced mass accretion rate to the initial mass accretion rate $\dot{M}_0 \approx 7 \times 10^{-7} M_\odot \text{yr}^{-1}$, as a function of the position of the pick-point of bumps, for azimuthal mode number $m = 1, 10, 15$, and 20 . The parameters of density bump are chosen as $\Sigma'_a = 0.01 \Sigma_0$ and $\Delta_1 = 0.1[R] = 10 \text{AU}$.